

## Surface modes and photonic modes in Casimir calculations for a compact cylinder

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2008 J. Phys. A: Math. Theor. 41 164005

(<http://iopscience.iop.org/1751-8121/41/16/164005>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.148

The article was downloaded on 03/06/2010 at 06:44

Please note that [terms and conditions apply](#).

# Surface modes and photonic modes in Casimir calculations for a compact cylinder

V V Nesterenko

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,  
Dubna 141 980, Russia

E-mail: [nestr@theor.jinr.ru](mailto:nestr@theor.jinr.ru)

Received 21 October 2007, in final form 5 January 2008

Published 9 April 2008

Online at [stacks.iop.org/JPhysA/41/164005](http://stacks.iop.org/JPhysA/41/164005)

## Abstract

A rigorous formulation of the problem of calculating the electromagnetic vacuum energy of an infinite dielectric cylinder is discussed. It is shown that the physically relevant spectrum of electromagnetic excitations includes the surface modes and photonic modes. The mathematical procedure of summing over this spectrum is proposed, and the transition to imaginary frequencies is accomplished. As a result, the imaginary-frequency representation for the vacuum energy is justified which has been used in previous Casimir studies for this configuration.

PACS numbers: 11.10.Gh, 42.50.Pq, 03.70.+k, 03.65.Sq, 11.30.Ly

## 1. Introduction

The notion of the elementary excitation spectrum is of paramount importance in all condensed matter physics [1, 2]. The excitations of different types result, as a rule, in different physical consequences. Therefore, it is of a certain interest for theoretical and experimental investigations of the Casimir effect to answer the question: what types of the electromagnetic oscillations are considered in the problem at hand [3–8]? Having elucidated this point one can hope to link, in a transparent way, the Casimir force with actual physical properties of the material boundaries. However, it is not easy to answer this question even when the Casimir force is calculated by making use of the familiar Lifshitz formula [9–11]. A rather complicated derivation of this formula [12] in the original papers initiated its obtaining anew by making use mainly of the mode summation method [12–19].

The Casimir calculations for nonflat boundaries turned out to be much more involved in comparison with those for planes [20]. Especially complicated calculations have been done for a circular cylinder [21–38]. It was also unclear that what kinds of electromagnetic excitations have been taken into account in these studies [39].

The present work seeks to propose a consistent derivation of the formula for the vacuum energy of electromagnetic field connected with a material cylinder by explicitly summing the contributions to this energy given by different branches of the electromagnetic spectrum in this problem.

The layout of the paper is as follows. In section 2, the spectral problem generated by the Maxwell equations for a compact infinite cylinder is formulated rigorously and the physically relevant spectrum of electromagnetic excitations for this configuration is determined. It is shown that this spectrum includes surface modes (bound states) and photonic modes. In section 3, the summation over this spectrum is accomplished by making use of the spectral density taking into account the photonic (continues) branch of the spectrum. In conclusion, section 4, the meaning of the obtained results is discussed briefly.

## 2. Physical spectrum of electromagnetic excitations for a cylinder

In the source-free case, the general solution to Maxwell equations can be represented in terms of two independent Hertz vectors [40]:

$$\mathbf{E} = \nabla \times \nabla \times \mathbf{\Pi}' + i\mu \frac{\omega}{c} \nabla \times \mathbf{\Pi}'', \quad (1)$$

$$\mathbf{H} = -i\varepsilon \frac{\omega}{c} \nabla \times \mathbf{\Pi}' + \nabla \times \nabla \times \mathbf{\Pi}''. \quad (2)$$

Here  $\mathbf{\Pi}'$  is the electric Hertz vector,  $\mathbf{\Pi}''$  is the magnetic Hertz vector,  $c$  is the velocity of light in vacuum, and the Gauss units are used. The Hertz vectors obey the Helmholtz vector equation

$$(\nabla^2 + k^2)\mathbf{\Pi} = 0, \quad (3)$$

where the wave number  $k$  is given by

$$k^2 = \varepsilon\mu \frac{\omega^2}{c^2}. \quad (4)$$

On the other hand, it is known that the general solution to Maxwell without sources equations can be derived from two scalar functions which may be chosen in deferent ways [41, 42]. For the configuration with cylindrical symmetry these functions can play the role of the axial components of the electric ( $\mathbf{\Pi}'$ ) and magnetic ( $\mathbf{\Pi}''$ ) Hertz vectors. The rest components of  $\mathbf{\Pi}'$  and  $\mathbf{\Pi}''$  are zero in this case. As a result, the Helmholtz vector equation (3) reduces to the scalar Helmholtz equations for  $\mathbf{\Pi}'_z \equiv \Pi'$  and  $\mathbf{\Pi}''_z \equiv \Pi''$

$$\left( \Delta + \varepsilon\mu \frac{\omega^2}{c^2} \right) \Pi = 0, \quad \Pi = \Pi', \Pi'' \quad (5)$$

with the following general solutions:

$$\Pi' = \sum_{n=0, \pm 1, \pm 2, \dots} a_n f_n^{\text{TM}}(r) e^{ihz + in\theta}, \quad (6)$$

$$\Pi'' = \sum_{n=0, \pm 1, \pm 2, \dots} b_n f_n^{\text{TE}}(r) e^{ihz + in\theta}. \quad (7)$$

The cylindrical coordinates  $(r, \theta, z)$  are used and the  $z$ -axis coincides with the axis of a circular infinite cylinder of radius  $a$ . The medium inside the cylinder has the permittivity  $\varepsilon_1$  and permeability  $\mu_1$ . These quantities outside the cylinder acquire the values  $\varepsilon_2$  and  $\mu_2$ , respectively. We assume for definiteness that the velocities of light inside and outside the

cylinder,  $c_1$  and  $c_2$ , respectively, obey the inequality  $c_1 < c_2$ , where  $c_s = c/(\varepsilon_s \mu_s)$ ,  $s = 1, 2$ . The wave vector along the  $z$ -axis is denoted by  $h$ . The amplitudes  $a_n$  and  $b_n$  for the solutions inside the cylinder will be denoted by  $a_n^i$  and  $b_n^i$ , respectively, and in the same way for solutions outside the cylinder we introduce the amplitudes  $a_n^e$  and  $b_n^e$ .

The functions  $f_n^{\text{TE}}(r)$  and  $f_n^{\text{TM}}(r)$  in the general solutions (6) and (7) obey the radial wave equation

$$\frac{d^2 f_n}{dr^2} + \frac{1}{r} \frac{df_n}{dr} + \left( k^2 - h^2 - \frac{n^2}{r^2} \right) f_n = 0, \quad f_n(r) = f_n^{\text{TE}}(r), \quad f_n^{\text{TM}}(r). \quad (8)$$

Inside the cylinder we put

$$f_n(r) = J_n(\lambda_1 r), \quad n = 0, 1, \dots, \quad 0 < r < a, \quad (9)$$

where  $J_n(z)$  is the Bessel function,  $\lambda_1 = \sqrt{k_1^2 - h^2}$ ,  $k_1^2 = \omega^2/c_1^2 = \varepsilon_1 \mu_1 \omega^2/c^2$ . Outside the cylinder we first consider ‘outgoing’ waves

$$f_n(r) = H_n^{(1)}(\lambda_2 r), \quad n = 0, 1, \dots, \quad r > a, \quad (10)$$

where  $H^{(1)}(z)$  is the Hankel function of the first kind,  $\lambda_2 = \sqrt{k_2^2 - h^2}$ ,  $k_2^2 = \omega^2/c_2^2 = \varepsilon_2 \mu_2 \omega^2/c^2$ .

In the radial solutions (9) and (10) the sign of  $\lambda_s^2 = k_s^2 - h^2$ ,  $s = 1, 2$ , is not fixed yet. Thus, in our consideration the solutions

$$f_n(r) = I_n(\tilde{\lambda}_1 r) \quad \text{for } r < a \quad (11)$$

and

$$f_n(r) = K_n(\tilde{\lambda}_2 r) \quad \text{for } r > a \quad (12)$$

are also admissible. Here  $\tilde{\lambda}_s^2 = h^2 - k_s^2$ ,  $s = 1, 2$ ,  $I_n(z) = i^{-n} J_n(iz)$  and  $K_n(z) = i^{n+1/2} H_n(iz)$  are the modified Bessel functions [43]. The sign of  $\lambda_s^2$ ,  $s = 1, 2$ , will be fixed below.

On the cylinder surface the matching conditions should be satisfied. These conditions require the continuity of tangential components of fields  $\mathbf{E}$  and  $\mathbf{H}$  when crossing cylinder surface

$$\text{discont}(\mathbf{E}_{\parallel}) = 0, \quad \text{discont}(\mathbf{H}_{\parallel}) = 0. \quad (13)$$

These are the conditions that couple the TE- and TM-solutions in the problem under study, when  $c_1 \neq c_2$ . Indeed, the matching conditions (13) give rise to a unique frequency equation determining admissible values of the spectral parameter  $\omega$  simultaneously for TE- and TM-solutions:

$$\frac{\omega^2 a^4}{c^2} \left( \varepsilon_1 \lambda_2 \frac{J'_n}{J_n} - \varepsilon_2 \lambda_1 \frac{H'_n}{H_n} \right) \left( \mu_1 \lambda_2 \frac{J'_n}{J_n} - \mu_2 \lambda_1 \frac{H'_n}{H_n} \right) - \frac{n^2 h^2 a^2}{\lambda_1^2 \lambda_2^2} \left[ \frac{\omega^2}{c^2} (\varepsilon_1 \mu_1 - \varepsilon_2 \mu_2) \right]^2 = 0, \quad n = 0, 1, 2, \dots \quad (14)$$

In this equation

$$J_n \equiv J_n(\lambda_1 a), \quad H_n \equiv H_n^{(1)}(\lambda_2 a), \quad \lambda_s = +\sqrt{\omega^2/c_s^2 - h^2}, \quad s = 1, 2, \quad (15)$$

the prime on the functions  $J_n$  and  $H_n$  means differentiation with respect to their arguments.

When  $c_1 = c_2$  the last term on the left-hand side of (14) vanishes and this equation splits into two independent equations which determine the eigenfrequencies for the TE- and TM-solutions separately. The same takes place for  $n = 0$ .

The roots of (14) are important in radio-engineering when developing the radio dielectric waveguides [44–50] and in fiber optics (optical waveguides [51, 52]). The results of the investigation of the frequency equation (14) determining the spectrum in the problem at hand can be summarized in the following way. All the *real roots* of this equation lie in the interval

$$c_1 h < \omega < c_2 h. \tag{16}$$

These roots make up two discrete sequences. In the interval (16), the frequency equation (14) can be rewritten in the form

$$\frac{\omega^2 a^4}{c^2} \left( \varepsilon_1 \bar{\lambda}_2 \frac{J'_n}{J_n} + \varepsilon_2 \lambda_1 \frac{K'_n}{K_n} \right) \left( \mu_1 \bar{\lambda}_2 \frac{J'_n}{J_n} + \mu_2 \lambda_1 \frac{K'_n}{K_n} \right) - \frac{n^2 h^2 a^2}{\lambda_1^2 \bar{\lambda}_2^2} \left[ \frac{\omega^2}{c^2} (\varepsilon_1 \mu_1 - \varepsilon_2 \mu_2) \right]^2 = 0, \tag{17}$$

$$n = 0, 1, 2, \dots$$

with the notation  $K_n \equiv K_n(\bar{\lambda}_2 a)$ .

When frequency  $\omega$  equals the real roots of equation (17), located in the interval (16), the ‘outgoing’ waves (10) become the functions decaying in the radial direction (12) (surface or evanescent waves). These eigenmodes describe the propagation of electromagnetic waves along the cylinder (waveguide solutions). The radial functions (11) are not realized in the problem under consideration.

Now we address the complex roots of the frequency equation (14). In the strip of the complex frequency plane

$$0 < \text{Re } \omega < c_2 h \tag{18}$$

there are no complex roots of (14) with  $\text{Im } \omega \neq 0$ .

In the semi-plane

$$\text{Re } \omega > c_2 h \tag{19}$$

for sure there are *complex roots* of (14) with  $\text{Im } \omega \neq 0$ . Indeed, in the domain (19) the left-hand side of (14) is a complex function of the complex variable  $\omega$ . The complex eigenfrequencies of a dielectric cylinder lead to leaky (radiating) modes. It is clear that these modes cannot carry the electromagnetic energy along the cylinder. For us it is important that the modes with complex  $\omega$  (quasi-normal modes [39]) do not satisfy standard completeness condition and as a result they cannot be used for quantization of electromagnetic field in the problem at hand.

In order to get rid of the complex eigenfrequencies and consequently to escape leaky or radiating modes we shall consider, outside the cylinder, the scattering states instead of outgoing waves. The scattering solutions to Maxwell equations can be derived from outgoing solutions by making use of the substitutions

$$\begin{aligned} a_n^e H_n^{(1)}(\lambda_2 r) &\rightarrow a_n^+ H_n^+(\lambda_2 r) + a_n^- H_n^-(\lambda_2 r) \\ b_n^e H_n^{(1)}(\lambda_2 r) &\rightarrow b_n^+ H_n^+(\lambda_2 r) + b_n^- H_n^-(\lambda_2 r). \end{aligned} \tag{20}$$

For simplicity in (20) the notations

$$H_n^+ \equiv H_n^{(1)}, \quad H_n^- \equiv H_n^{(2)} \tag{21}$$

are introduced.

As a result, for a given  $n$  and  $h$  we have six amplitudes  $a_n^i, b_n^i, a_n^+, a_n^-, b_n^+, b_n^-$ . The matching conditions at the cylinder surface lead to four linear homogeneous equations for these amplitudes. Hence no restrictions arise here for the spectral parameter  $\omega^2/c^2$ .

Eliminating in these equations the amplitudes  $a_n^i$  and  $b_n^i$  we are left with two equations in four amplitudes  $a_n^\pm$  and  $b_n^\pm$ :

$$K^- \begin{pmatrix} a_n^+ \\ b_n^+ \end{pmatrix} = K^+ \begin{pmatrix} a_n^- \\ b_n^- \end{pmatrix}, \quad K^\pm = \mp \begin{pmatrix} \alpha^\mp & \beta^\mp \\ \gamma^\mp & \alpha^\mp \end{pmatrix}, \tag{22}$$

where

$$\begin{aligned} \alpha_n^\pm &= -\frac{nh}{a} \left( 1 - \frac{\lambda_2^2}{\lambda_1^2} \right), \\ \beta_n^\pm &= -i\frac{\omega}{c} \left( \mu_2\lambda_2 H_n^{\pm'} - \mu_1\lambda_1 \frac{J_n' \lambda_2^2}{J_n \lambda_1^2} H_n^\pm \right), \\ \gamma_n^\pm &= i\frac{\omega}{c} \left( \varepsilon_2\lambda_2 H_n^{\pm'} - \varepsilon_1\lambda_1 \frac{J_n' \lambda_2^2}{J_n \lambda_1^2} H_n^\pm \right). \end{aligned} \quad (23)$$

The  $S$  matrix in this problem

$$\begin{pmatrix} a_n^+ \\ b_n^+ \end{pmatrix} = S \begin{pmatrix} a_n^- \\ b_n^- \end{pmatrix} \quad (24)$$

obeys obviously the following matrix equation:

$$K^- S = K^+, \quad (25)$$

and

$$\det S = \frac{\det K^+}{\det K^-}. \quad (26)$$

By a direct calculation one can easily show that  $\det K^-$  coincides (up to unimportant multiplier) with the left-hand side of the frequency equation (14). Thus, this equation can be rewritten in the form

$$\det K^- = 0. \quad (27)$$

Surprisingly formulae (26) and (27) for the  $S$  matrix were not known in the literature devoted to the electromagnetic scattering by a cylinder [54, 55].

Summarizing we infer that the spectrum of electromagnetic oscillations in the problem under study consists of discrete values  $\omega_{n\alpha}$ ,  $c_1 h < \omega_{n\alpha} < c_2 h$ , corresponding to the surface modes and a continuous branch of the spectrum with real positive  $\omega$ ,  $c_2 h < \omega < \infty$ . In mathematical scattering theory [53], it is proved that the bound states and scattering states form together a complete set.

It is worth noting here that there is no complete analogy between analytic properties of the scattering matrix (or the Jost function) in the problem under study and in the standard theory of potential scattering [53]. Indeed, in the case of a scalar potential scattering we have, instead of frequency equation (14) or (27), the requirement of vanishing the Jost function  $F(-k)$ . The bound states in potential scattering lead to pure imaginary zeros of this function. On the real axes  $k$  this function has no zeros. At first sight we have here a contradiction. However it is not the case. The point is that the bound states (or surface modes) in the Casimir calculations are due to the appearance of the longitudinal wave vector  $h$  in these studies. In the standard potential scattering there is no such a vector. This vector is also absent in the Casimir calculations for a dielectric ball, and as a consequence in the latter problem there are no surface modes (bound states). All this implies in particular that the analytical properties of the scattering matrix in the Casimir calculations should be revealed by a direct analysis of its explicit form without referring to the potential scattering.

### 3. Summation over the spectrum and transition to imaginary frequencies

Now we address the calculation of the vacuum energy in the problem at hand proceeding from the standard mode-by-mode summation

$$E_c = \frac{1}{2} \sum_{\{q\}} \omega_q = \int_{-\infty}^{\infty} \frac{dh}{2\pi} \sum_{n=0}^{\infty} \left[ \sum_{\alpha} \omega_{n\alpha}(h) + \int_{c_2 h}^{\infty} \omega \Delta \rho_n(\omega, h) d\omega \right], \quad (28)$$

where the prime over the sum sign means that the term with  $n = 0$  is taken with the weight  $\frac{1}{2}$ . The first term in square brackets is responsible for the surface waves contribution and the second one describes the contribution of the photonic modes. The latter contribution is represented by making use of the respective spectral shift function  $\Delta\rho$  [39]. The rigorous mathematical scattering theory gives the following expression for the spectral density shift:

$$\Delta\rho(k) \equiv \rho(k) - \rho_0(k) = \frac{1}{2\pi i} \frac{d}{dk} \operatorname{tr} \ln S(k) = \frac{1}{2\pi i} \frac{d}{dk} \ln \det S(k). \quad (29)$$

Here,  $\rho(k)$  is the density of states for a given potential (or for a given boundary condition in the case of compound media) and  $\rho_0(k)$  is the spectral density in the respective free spectral problem (for vanishing potential or for homogeneous unbounded space). It is obvious that in the Casimir calculations one has to use just  $\Delta\rho(k)$  subtracting at this point the so-called Minkowski spacetime contribution to the vacuum energy.

In the case of scalar scattering problem the Jost functions  $f(k)$  and  $f(-k)$ , the scattering matrix  $S(k)$  and the phase shift  $\delta(k)$  are related by the formula

$$S(k) = e^{2i\delta(k)} = \frac{f(k)}{f(-k)}. \quad (30)$$

Substitution of (30) into (28) gives more familiar formula for spectral density [56]:

$$\Delta\rho(k) = \frac{1}{2\pi i} \frac{d}{dk} \ln \frac{f(k)}{f(-k)} = \frac{1}{\pi} \frac{d}{dk} \delta(k). \quad (31)$$

In the problem under consideration the TE and TM modes do not decouple. Therefore we are dealing here with the matrix  $(2 \times 2)$  scattering problem, and we must use the spectral density defined by (29).

The contribution of the surface modes in (28) can be represented by the contour integral

$$\sum_{\alpha} \omega_{n\alpha} = \frac{1}{2\pi i} \oint_C \omega \frac{d}{d\omega} \ln F_n(\omega) d\omega, \quad (32)$$

where  $F_n(\omega)$  is the left-hand side of (17). This equation was written for real  $\omega$ . However, in the contour integral (32) an analytical continuation of this function to the complex frequency plane should be used. It can be done immediately in terms of  $\det K^+$  (lower semi-plane  $\omega$ ) and  $\det K^-$  (upper semi-plane  $\omega$ ). After that we can use for both terms in (28) the contour integral representations with the contours  $C_+$  and  $C_-$ , respectively. The contour  $C_-$  starts at  $i\infty$  and goes along the positive imaginary axis to the origin and after that it goes along the positive real semi-axis to infinity. The contour  $C_+$  is obtained by the reflection of  $C_-$  to the upper semi-plane  $\omega$ . As a result, we arrive at the following imaginary-frequency representation of the vacuum energy in the problem at hand:

$$E_c = \int_{-\infty}^{\infty} \frac{dh}{2\pi} \sum'_{n=0} \int_0^{\infty} y \frac{d}{dy} \ln F_n(iy, h) dy, \quad (33)$$

where  $F_n(\omega, h)$  is the left-hand side of the frequency equation (14). It is this representation that has been used in the Casimir calculations for a material cylinder.

#### 4. Conclusion

We have shown that in the case of a material cylinder there are two types of electromagnetic excitations which are physically relevant: (i) surface modes and (ii) photonic modes. For a consistent transition to imaginary frequencies both the branches of the spectrum are to be taken into account. The contribution to the Casimir energy due to the surface modes and photonic

modes can be separated only in terms of real frequencies. Upon transition to imaginary frequencies these contributions are indivisible. Presented consideration rigorously justifies the imaginary-frequency representation for the Casimir energy of a compact infinite cylinder that has been used in many previous papers dealing with the investigation of this energy.

It should be noted that the mathematical consideration presented here is completely applicable to the Lifshitz configuration, namely, to an infinite dielectric plate placed in vacuum (dielectric films).

## Acknowledgments

This study has been accomplished by the financial support of the Russian Foundation for Basic Research (Grant No 06-01-00120) and the Heisenberg–Landau Program. VNN would like to thank the organizer Michael Bordag for the excellent workshop on Quantum Field Theory Under the Influence of External Conditions held in Leipzig in September 2007 (QFEXT07) and many participants in this workshop for illuminating lectures and discussions.

## References

- [1] Lifshitz E M and Pitaevskii L P 1980 *Landau and Lifshitz Course of Theoretical Physics: Statistical Physics* Part 2 (Washington: Butterworth-Heinemann)
- [2] Kittel Ch 1995 *Introduction to Solid State Physics* 7th edn (New York: Wiley)
- [3] Ford L H 1998 *Phys. Rev. D* **38** 528
- [4] Ford L H 1993 *Phys. Rev. A* **48** 2962
- [5] Henkel C, Joulain K, Mulet J-Ph and Greffet J-J 2004 *Phys. Rev. A* **69** 023808
- [6] Genet C, Intraiva F, Lambrecht A and Reynaud S 2004 *Ann. Fond. L. de Broglie* **29** 311 (Preprint [quant-ph/0302072](#))
- [7] Intraiva F and Lambrecht A 2005 *Phys. Rev. Lett.* **94** 110404
- [8] Bordag M 2006 *J. Phys. A: Math. Gen.* **39** 6173
- [9] Lifshitz E M 1965 *Zh. Eksp. Teor. Fiz.* **29** 94  
Lifshitz E M 1956 *Sov. Phys.—JETP* **2** 73 (Engl. Transl.)
- [10] Dzyaloshinskii I E, Lifshitz E M and Pitaevskii L P 1959 *Zh. Eksp. Teor. Fiz.* **37** 229  
Dzyaloshinskii I E, Lifshitz E M and Pitaevskii L P 1960 *Sov. Phys.—JETP* **10** 161 (Engl. Transl.)
- [11] Dzyaloshinskii I E, Lifshitz E M and Pitaevskii L P 1961 *Adv. Phys.* **10** 165  
Dzyaloshinskii I E, Lifshitz E M and Pitaevskii L P 1961 *Usp. Fiz. Nauk* **73** 381  
Dzyaloshinskii I E, Lifshitz E M and Pitaevskii L P 1961 *Sov. Phys.—Usp.* **4** 153 (Engl. Transl.)
- [12] Spruch L and Tikochinsky Y 1993 *Phys. Rev. A* **48** 4213
- [13] Tikochinsky Y and Spruch L 1993 *Phys. Rev. A* **48** 4223
- [14] van Kampen N G, Nijboer B R A and Schram K 1968 *Phys. Lett. A* **26** 307
- [15] Ninham B W, Parsegian V A and Weiss G H 1970 *J. Stat. Phys.* **2** 323
- [16] Gerlach E 1971 *Phys. Rev. B* **4** 393
- [17] Schram K 1973 *Phys. Lett. A* **43** 282
- [18] Langbein D 1973 *Solid State Commun.* **12** 853
- [19] Klimchitskaya G L, Mohideen U and Mostepanenko V M 2000 *Phys. Rev. A* **61** 062107
- [20] Nesterenko V V, Lambiase G and Scarpetta G 2004 *Riv. Nuovo Cimento* **27** 1–74 (Preprint [hep-th/0503100](#))
- [21] DeRaad L L Jr and Milton K 1981 *Ann. Phys., NY* **136** 229
- [22] Brevik I and Nyland G H 1994 *Ann. Phys., NY* **230** 321
- [23] DeRaad L L Jr 1985 *Fortschr. Phys.* **33** 117
- [24] Milton K A, Nesterenko A V and Nesterenko V V 1999 *Phys. Rev. D* **59** 105009 (Preprint [hep-th/9711168](#))
- [25] Lambiase G, Nesterenko V V and Bordag M 1999 *J. Math. Phys.* **40** 6254 (Preprint [hep-th/9812059](#))
- [26] Nesterenko V V and Pirozhenko I G 2000 *J. Math. Phys.* **41** 4521 (Preprint [hep-th/9910097](#))
- [27] Nesterenko V V and Pirozhenko I G 1999 *Phys. Rev. D* **60** 125007 (Preprint [hep-th/9907192](#))
- [28] Bordag M and Pirozhenko I G 2001 *Phys. Rev. D* **64** 025019
- [29] Caverio-Peláez I and Milton K A 2005 *Ann. Phys., NY* **320** 108 (Preprint [hep-th/0412135](#))
- [30] Caverio-Peláez I and Milton K A 2006 *J. Phys. A: Math. Gen.* **39** 6225 (Preprint [hep-th/0511171](#))
- [31] Caverio-Peláez I, Milton K A and Kirsten K 2007 *J. Phys. A: Math. Theor.* **40** 3607 (Preprint [hep-th/0607154](#))



- [32] Barton G 2001 *J. Phys. A: Math. Gen.* **34** 4083
- [33] Gosdzinsky P and Romeo A 1998 *Phys. Lett. B* **441** 265 (Preprint [hep-th/9809199](#))
- [34] Klich I and Romeo A 2000 *Phys. Lett. B* **476** 369 (Preprint [hep-th/9912223](#))
- [35] Romeo A and Milton K A 2005 *Phys. Lett. B* **621** 309 (Preprint [hep-th/0504207](#))
- [36] Romeo A and Milton K A 2006 *J. Phys. A: Math. Gen.* **39** 6703
- [37] Brevik I and Romeo A 2007 *Phys. Scr.* **76** 48 (Preprint [hep-th/0601211](#))
- [38] Schaden M 2006 Semiclassical electromagnetic Casimir self-energies Preprint [hep-th/0604119](#)
- [39] Nesterenko V V 2006 *J. Phys. A: Math. Gen.* **39** 6609 (Preprint [hep-th/0511018](#))
- [40] Stratton J A 1941 *Electromagnetic Theory* (New York: McGraw-Hill)
- [41] Whittaker E T 1904 *Proc. Lond. Math. Soc.* **1** 367
- [42] Nisbet A 1955 *Proc. R. Soc. A* **231** 250
- [43] Gradshteyn I S and Ryzhik I M 2000 *Table of Integrals, Series, and Products* 6th edn (New York: Academic)
- [44] Rayleigh L 1897 *Phil. Mag. J. Sci.* **43** 125
- [45] Hondros D 1909 *Ann. Phys.* **30** 905
- [46] Hondros D 1909 *Phys. Z.* **10** 804
- [47] Hondros D and Debye P 1910 *Ann. Phys.* **32** 465
- [48] Schelkunoff S A 1943 *Electromagnetic Waves* (New York: Van Nostrand-Reinhold)
- [49] Katzenelenbaum B S 1949 *Zh. Techn. Fiz.* **19** 1168, 1182 (in Russian)
- [50] Borgnis E and Papas C H 1958 Electromagnetic waveguides and resonators *Encyclopedia of Physics* vol XVI ed S Flüge (Berlin: Springer) pp 285–422
- [51] Snitzer E 1961 *J. Opt. Soc. Am.* **51** 491
- [52] Marcuse D 1974 *Theory of Dielectric Optical Waveguides* (New York: Academic)
- [53] Newton R G 2002 *Scattering Theory of Waves and Particles* (New York: Dover)
- [54] Wait J R 1955 *Canad. J. Phys.* **33** 189
- [55] van de Hulst H C 1957 *Light Scattering by Small Particles* (New York: Wiley)
- [56] Landau L D and Lifshitz E M 1996 *Statistical Physics Part 1* 3rd edn (Oxford: Butterworth-Heinemann)